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The permittivity of a planar percolation system

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Abstract. We have studied the permittivity of a thin (planar) percolation system (PPS) spread over a plane which is in turn sunk into the host medium. We present the numerical simulation of the PPS as well as RG speculations. It appears that u and s indices $\left(\varepsilon(L, p_c, \omega = 0) \sim L^{s/\nu}, \varepsilon(L = \infty, p_c, \omega) \sim \omega^{-(1-u)}\right)$ of the planar and 2D systems are different: $\Delta(s/\nu) \simeq 0.25$ and $u_{PPS} \simeq 0.42 < u_{2D} = 0.5$. We find out that the Shklovskii–Efros scaling relation is true in spite of the fact that the PPS is a non-universal system. The indices depend upon the permittivity of the host medium.

The object of this investigation is the so-called planar percolation system (PPS) which is a metal–insulator mixture spread over a plane that is in turn sunk into a uniform insulating medium (figure 1). Examples of such systems are thin (island) films and thin composite (granular) films. Recently more attention has been paid to properties of thin films, which are widely used in optics. To date there is no clear physical description of certain absorption and reflection features of the systems (see [1] and [2]) and references therein). There are similar difficulties in interpreting the absorption properties of multilayer coatings built up of thin composite (granular) sheets. We attempt to clear up the situation.

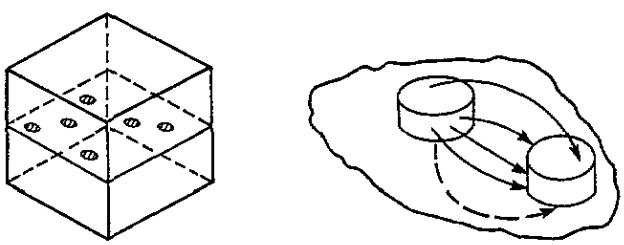

Figure 1. An example of a planar system. All the inclusions, which are actually metallic spots, are lying in one plane. Above and below this plane there exists an insulating medium. If the electric field is applied along the plane some of the force lines should lie in the plane but some should leave this plane starting at the top (bottom) of one inclusion and return through the top (bottom) of the other inclusion. Thus the pattern of force lines is three dimensional.

Efros and Shklovskii [3] were the first to point out that the PPS and two-dimensional (2D) systems were different if one were interested in the effective permittivity ε_{eff}. 2D systems are systems in which properties are independent of one of the coordinates, say z. An example of a 2D system is a set of metallic cylinders immersed in a dielectric matrix with their axes parallel to z (figure 2). Thus the inclusions in the planar system differ from the 2D case in

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having top and bottom surfaces, as in the 3D case (figure 3). The difference between the planar system and the 3D system is that all inclusions in the planar system lie in one plane (figures 1, 3). The force line schemes illustrating the difference between these systems are also presented in figures 1–3. All the force lines in the 2D system are perpendicular to the z axis while in the planar systems and in the 3D systems this is not the case.

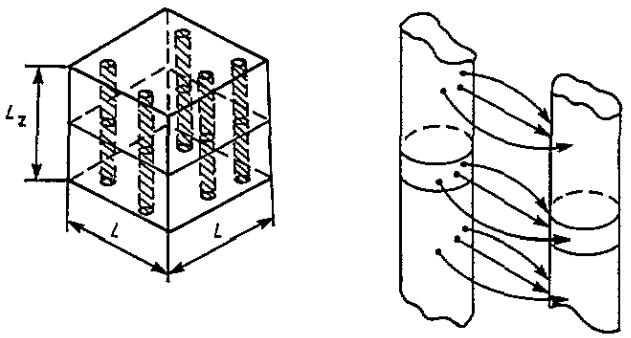

Figure 2. A 2D system is a system having properties independent of one of the coordinates, say z. If an external electric field is applied perpendicular to the z axis all force lines should be perpendicular to the z axis too. Thus we have a 2D pattern of force lines.

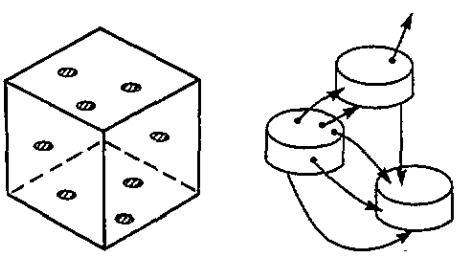

Figure 3. In a 3D system all inclusions are distributed at random. Independent of the direction of the applied electric field, the pattern of force lines is 3D.

To compare the 2D system with the planar system it is convenient to introduce the linear capacity $C^l(L_z) = C/L_z$ where C is the capacity of the system and L_z is thickness of the system. C^l is a quantity that is linearly proportional to the effective permittivity ε_{eff}. For a planar system $L_z = a_0$, where a_0 is the average diameter of an inclusion. The linear capacity C^l_{PPS} is defined as

$$C^l_{\text{PPS}} = C/a.$$

For the 2D system we should consider a limit process $L_z \to \infty$. Figure 2 presents an $(L \times L \times L_z)$ system, which as $L_z \to \infty$ becomes a finite $(L \times L)$ 2D system. For the finite $(L \times L)$ 2D system one can introduce the linear capacity

$$C^l_{2D} = \lim_{L_z \to \infty} C^l(L_z) = \lim_{L_z \to \infty} CL/(L \times L_z).$$

It is well known [3, 6] that near the percolation threshold p_c the value of ε_{eff} is independent of L if L is greater than the correlation length ξ; in the opposite case

$\varepsilon_{\text{eff}}(L) \sim L^{s/\nu}$ [3], where $s_{3D} \simeq 0.8$ [4, 5], $s_{2D} \simeq 1.33 \pm 0.02$ [6], and ν is the correlation length index ($\nu_{2D} = \frac{4}{3}$ [6], $\nu_{3D} \simeq 0.88$ [4]). It is obvious that ε_{eff} differs from unity by virtue of the linear capacities

$$C_{\text{cl}}^{\text{l}} \simeq \lim_{L_z \to \infty} (\mathcal{L} L_z / a)/L_z \simeq \mathcal{L}/a \tag{1}$$

between neighbouring metallic clusters where $\mathcal{L}$ is a shared perimeter of these two clusters. We consider the distance between neighbouring clusters to be equal to a_0. At the percolation threshold the average value of $\mathcal{L}$ is proportional to L^y where y is some critical index.

Let us come back to the PPS and consider a finite $(L \times L \times a)$ system. Calculating the linear capacity C_{cl}^{l} of two metallic clusters in such a system one has to take into account surface charges on the top and bottom surfaces of inclusions. To estimate the capacity it is reasonable to consider a cluster as a metallic disc of thickness $L_z = a$ and ascribe to the disc radius the value of the cluster gyration radius $r_g \sim L^x$ [7]. The problem in evaluating the capacity between neighbouring discs is thoroughly considered in [8]. It has a solution in the form of Kirchhoff's well known correction formula for a capacitor and gives, due to edge effects on the value (1) for the linear capacity C_{cl}^{l}, the extra logarithmic term

$$\Delta C_{\text{cl}}^{\text{l}} \simeq \left[\frac{\mathcal{L}}{8\pi^2} \ln\left(\sqrt{r_g \mathcal{L}}/a\right) \right] / L_z. \tag{2}$$

In the case of the PPS we shall take $L_z \simeq a$. Comparing (1) and (2) one would expect only a logarithmic correction to the power law $\varepsilon_{\text{PPS}}(L) \sim L^{s/\nu} \ln L$. The same result ($s_{\text{PPS}} \simeq s_{2D}$) can be easily obtained with the real-space renormalization group (RG) scheme proposed by Bernasconi [9]. In terms of a conventional bond problem† (see for example [5] and [10]) the planar system is imitated by one of the layers in the cubic network (see [11] and figure 1). Following [9] a block of $b \times b$ bonds belonging to this layer is substituted by a Wheatstone bridge (see figure 4). To account for the connection between two points of the PPS through the ambient medium it seems reasonable at the nth step of the RG transformation to consider the extra bonds with conductance $g_{\text{h}}^{(n)}$ placed in parallel to each non-superconducting bond of the PPS (figure 4). The value of $g_{\text{h}}^{(n)}$ may be chosen to be equal to the conductance between two points in host space separated by a single renormalized bond. This value is equal to $3\sigma_{\text{h}}^{(n)} = 3\sigma_h b^n$ [10], where σ_{h} and $\sigma_{\text{h}}^{(n)} = \sigma_{\text{h}} b^n$ are the conductance of the original and renormalized bonds of the host medium. As the size of the system increases the conductance of the renormalized 'metallic' bond lying on the plane also increases, governed by Bernasconi's RG equation [9]

$$\sigma_{\text{m}}^{(n+1)} = \lambda_\sigma \sigma_{\text{m}}^{(n)}$$

where $\lambda_\sigma = b^{s_{2D}/\nu}$. For the PPS, instead of a single RG equation we obtain a system of RG equations:

$$\sigma_{\text{PPS}}^{(n+1)} = \lambda_\sigma \sigma_{\text{PPS}}^{(n)} + \lambda_\sigma g_{\text{h}}^{(n)} \qquad g_{\text{h}}^{(n+1)} = b g_{\text{h}}^{(n)}$$

† Below, we study a related problem of a metal–superconductor mixture sunk into a host medium with conductivity σ_{h}. The conductivity of metallic inclusions is σ_{m} and that of superconducting ones is σ_{s}. The case $\sigma_{\text{s}} = \infty$ from a mathematical point of view is completely adequate for the case of a metal–dielectric mixture in the $\omega = 0$ limit [5, 10] but substituting σ for ε and j for D. For convenience, at a finite frequency $\omega \neq 0$ we also treat the system in terms of the effective conductivity σ_{eff} which is connected with ε_{eff} by a simple relation: $\sigma_{\text{eff}} = i\omega\varepsilon_{\text{eff}}/4\pi$. In our consideration this corresponds to the case with finite σ_{s}.

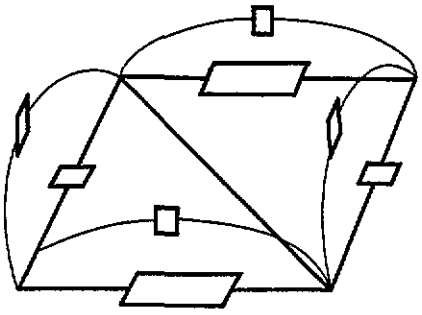

Figure 4. A block of $(b\times b\times b)$ bonds including $(b\times b)$ bonds of the PPS is presented as a Wheatstone bridge for $(b\times b)$ PPS bonds and the extra bonds with conductance $g_h^{(n)}$ placed in parallel with each non-superconducting bond of the PPS.

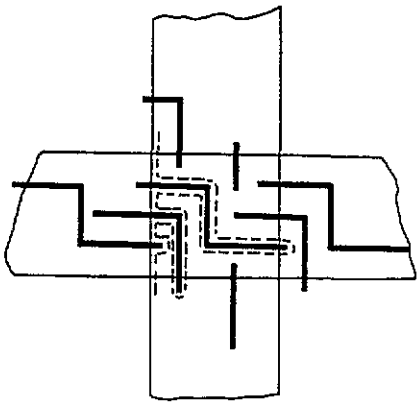

Figure 5. The cutting surface splits the system in two and avoids crossing any superconducting bond. The current through this surface determines the whole conductance G.

with eigenvalues $\lambda_\sigma = b^{s_{2D}/\nu} \simeq b$ and b. The value of s_{PPS}/ν is determined by the largest eigenvalue. Thus

$$|s_{PPS} - s_{2D}| \leqslant (\nu_{2D} - s_{2D}). \tag{3}$$

The $(\nu_{2D} - s_{2D})$ difference is not more than 0.02 [6]. The case $\nu_{2D} = s_{2D}$ corresponds to the logarithmic correction for the power law.

On the other hand, to explain the experimental data of [2] Coniglio *et al* [11] treated the system as a thick $(L \times L \times L)$ 3D object and declared new values of critical indices s and u different from those for the 2D case. The index u describes the frequency dependence of $\varepsilon_{eff} \sim \omega^{-(1-u)}$ ($u_{2D} = 0.5$ [6]). Furthermore, the authors predicted u to be near unity as well as breaking the Shklovskii–Efros scaling relation $u = t/(s+t)$ between indices [3].

To clear up the situation we carried out the numerical simulation of the PPS. In terms of the bond problem we study the interaction of an electromagnetic field with the PPS in the static ($\omega = 0$) case, which corresponds to the case $\sigma_s = \infty$, as well as in the quasi-static case ($\lambda = 2\pi c/\omega \gg \min L, \xi$) which corresponds to the case of finite σ_s †. We simulate the planar system by one of the layers in a uniform $(L \times L \times L)$ lattice (figure 1). To estimate the value of the critical index s we explore the dependence of the effective conductivity σ_{PPS} on the size L of the system at $\sigma_s = \infty$ and $\sigma_h = \sigma_m$. The concentration of superconducting bonds in the metal–superconductor layer is equal to p_c. There is an obvious relation between the whole conductance G of the $(L \times L \times L)$ system and the value of σ_{PPS}:

$$G = (L-1)\sigma_h + \sigma_{PPS}. \tag{4}$$

The main task of the computer simulation is to calculate the potentials ϕ of the nodes of the $(L \times L \times L)$ cubic network. First of all we extract superconducting clusters by Tarjan's algorithm [12]. Regarding each superconducting cluster as one node (all sites of the superconducting cluster have the same potential) we avoid the iteration procedure $h = \sigma_m/\sigma_s \to 0$ and solve the direct problem $\sigma_m = 1$, $\sigma_s = \infty$.

To diminish the influence of boundaries we apply periodic boundary conditions not only in the directions perpendicular to the applied field but also in the parallel direction:

$$\phi(x, y, z) = \phi(x, y, z + L) = \phi(x, y + L, z) = \phi(x + L, y, z) - U.$$

Thus, we eliminate not only dielectric walls around a sample but also the superconducting plates usually used (see for example [10]) to apply the external voltage U. These plates would strongly disturb the system. In our case only the external electric field $E = U/L$ is applied to the system parallel to the x axis.

In the static case ($\sigma_s = \infty$) we have a problem: how to calculate G. Indeed, the currents through superconducting bonds are left completely undetermined and hence the current through any plane x = constant is undetermined too. To settle this problem we extract the so-called cutting surface (see figure 5). This surface splits the system in two without crossing any superconducting bond. The current through this surface determines the whole conductance G. For each random realization of the system we calculate the conductivity of the PPS ($\sigma_h = \sigma_m$) and the conductivity of the corresponding 2D system ($\sigma_h = 0$). These turn out to be different. At the percolation threshold the ratio σ_{PPS}/σ_{2D} behaves as $L^{s''/\nu}$ with $s''/\nu = 0.25 \pm 0.01$, which agrees with the result obtained in [11] and contradicts both (2) and (3). The results of our computer simulation are presented in figure 6. Thus

$$s_{2D} < s_{PPS} = s_{2D} + s''.$$

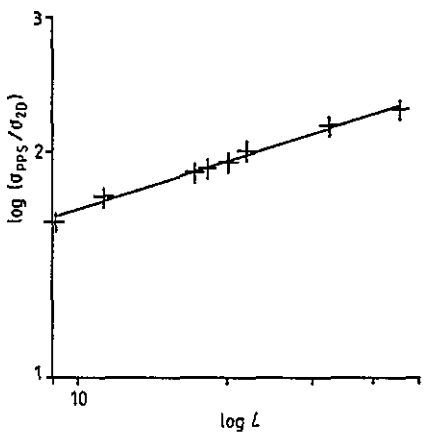

Figure 6. The dependence of the ratio σ_{PPS}/σ_{2D} on the system size L. By calculating the ratio we consider the same random configuration for both the PPS and the 2D system and then average over the sequence of 300 realizations.

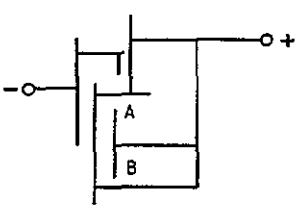

Figure 7. An example of a configuration that makes a different contribution to the capacity of the PPS and to the capacity of the 2D system. The part AB of the cluster is completely hidden in the 2D system but takes part in the production of capacity in the PPS.

The key to this inequality is that there are some parts of the clusters that do not make any contribution to the whole capacity for the 2D system but do for the PPS (see figure 7). These effects are closely connected with the planarity of the graph of an infinite cluster and cannot be observed for all $(d-1)$-hyper-plane systems, but only for $d = 3$. To verify this assumption we perform the computer simulation for $d = 2$. The percolation 1-hyper-plane

system (linear system) is a 1D percolation system lying on the plane of the host medium. This linear system may be thought of as a system of parallel strips lying in one plane. The field is applied parallel to the plane and perpendicular to the strips. To calculate the conductivity we use the transfer matrix algorithm [13]. The exponent of the linear system appears to be equal to that of the 1D system, which confirms our assumption.

To calculate the effective conductivity σ_{PPS} in the quasi-static case we use the well known transfer matrix method [11, 13]. The only dissimilarity is that we consider periodic boundary conditions along the applied field. In other aspects the scheme of the simulation is the same as in the static case but the ratio $h = \sigma_h/\sigma s$ is finite. At high values of h (see figure 8) σ_{PPS} behaves as $h^{u_{PPS}}$ with the value of $u_{PPS} = 0.420 \pm 0.04$, which is near the Efros and Shklovskii [3] prediction $u_{PPS} = t/(s_{PPS} + t) \simeq 0.44 \pm 0.10$. The index $s_{PPS} = s_{2D} + s''$ is a new PPS index. We use for t its 2D value $t_{2D} = \nu_{2D}$ [16]. Indeed, if the host medium is an insulator and L_z is equal to the size of the metallic inclusion, the index t describing the conductivity of a metal–insulator mixture above p_c is the same for both planar and 2D systems.

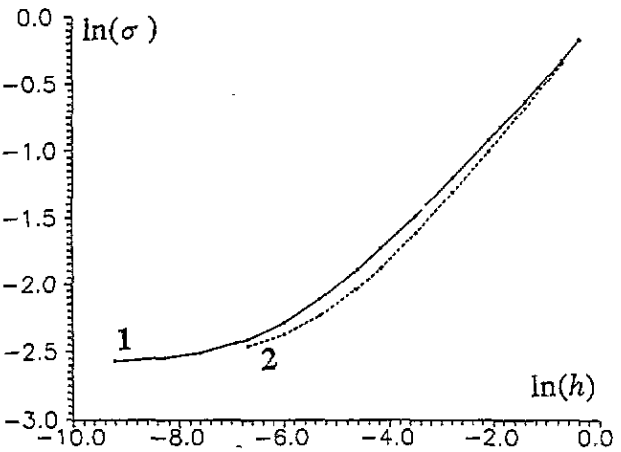

Figure 8. The quasi-static case. The dependence of the effective conductivity of the system on the ratio $h = \sigma_m/\sigma_s$. Curve 1 represents σ_{PPS}, curve 2 σ_{2D}. The systems produced during transfer matrix simulation are of size (20 × 20 × 5000).

The frequency dependence of $\varepsilon_{PPS} \sim \omega^{-0.58}$ is stronger than that of the 2D system, contrary to the statement of Coniglio *et al* [11] who predicted that $(1 - u)$ is near zero. This visible disagreement is not a consequence of a discrepancy in computer simulation results but is due to different interpretation of the results. Coniglio *et al* [11] applied the scaling hypothesis to the whole conductance G of an $(L \times L \times L)$ system. They assert

$$G \sim \begin{cases} \sigma_s L^{-t/\nu} & \sigma_m = 0 \quad (5a) \\ \sigma_m L^{1+s''/\nu} & \sigma_s = \infty \quad (5b) \\ \sigma_m^u \sigma_s^{(1-u)} L & \sigma_m \text{ and } \sigma_s \text{ finite.} \quad (5c) \end{cases}$$

From (4) we can see that in the limits $\sigma_s \neq \infty$ and $\sigma_h = \sigma_m = 0$ the whole conductance G is equal to the PPS conductance or conductivity, which is exactly the same. The contribution of the remaining host medium is negligible. In the $\sigma_s = \infty$ limit the same behaviour follows

from (4) and the asymptotic ratio $(\sigma_h L)/\sigma_{PPS} \sim L^{-(s_{PPS}/\nu-1)} \to 0$ as $L \to \infty$. Hence, the results obtained in this work agree with ($5a_0$) and ($5b$). At conditions corresponding to ($5c$), σ_{PPS} according to scaling is proportional to $\sigma_m^{u_{PPS}}\sigma_s^{(1-u_{PPS})}$ and is a finite quantity independent of L. Using (4) we obtain in the $L \to \infty$ limit that $G \simeq \sigma_m L$. This expression formally agrees with ($5c$) at $u = 1$. This high value of $u \gg t/(s+t)$ is due to shunting of the planar system by the ambient host medium and only denotes that the value of σ_{PPS} is finite, saying nothing about its value.

Besides the scaling relationship between indices we observe another behaviour of σ_{PPS} at $h \to 0$: if there is no percolation channel in the system then $\sigma_{PPS} \sim h$, otherwise $\sigma_{PPS} \sim \sigma_{2D}$ (see figure 8). The observed cross-over illustrates the standard finite scaling behaviour of $\sigma_{PPS} = \sigma_s h^u f(L/\xi(h))$ with

$$f(x) \propto \begin{cases} x^{s/\nu} & x \to 0 \\ 1 & x \to \infty \end{cases} \tag{6}$$

where $\xi(h) \sim h^{-\nu/(t+s)}$ is a correlation length at finite h. At $(L/\xi(h)) \gg 1$ we have $\sigma_{PPS} \sim h^{u_{PPS}}$ and at $(L/\xi(h)) < 1$ we have $\sigma_{PPS} \sim \sigma_m \Psi(L)$. To find the dependence $\Psi(L)$ we consider the $(L \times L \times M)$ system. The computer simulation shows that as M varies from unity to infinity σ_{PPS} changes from its 2D value to some finite value (see figure 9). $\Psi(L)$ respectively changes from $L^{-s_{2D}/\nu}$ to $L^{-s_{PPS}/\nu}$. There is a corresponding correlation length $\xi_\perp \simeq L$ determining the thickness of the system starting with which we observe macroscopic properties independent of M.

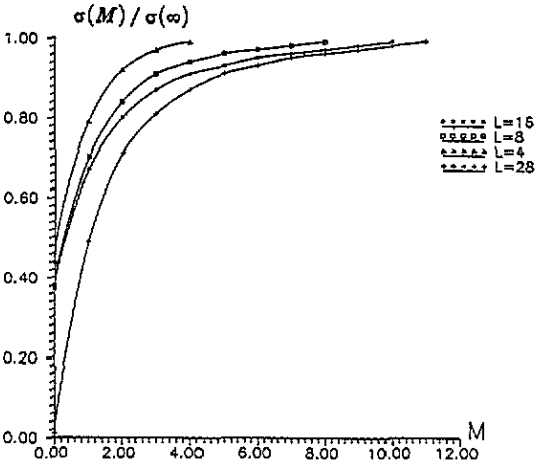

Figure 9. The dependence of $\sigma_{PPS}(M, L)/\sigma_{PPS}(\infty, L)$ on the period M in the direction perpendicular to the plane of the PPS for fixed values of L: Δ, L =4; □, L = 8; ⋆, L = 16; +, L = 24.

If $\sigma_h \neq \sigma_m$, σ_{PPS} depends on both σ_m and σ_h. The dependence can be formally rewritten in the form $\sigma_{PPS} = \sigma_s h^{u_{PPS}} f(\tau, h, h\sigma_h/\sigma_m)$. As σ_h tends to zero σ_{PPS} goes to σ_{2D}, if $\sigma_h = \sigma_m$ we obtain the previous results. Our computer simulation shows that the exponent s_{PPS} depends upon the conductivity σ_h of the host medium (figures 10 and 11) and as a consequence it is not universal. It is bounded by

$$s_{2D} < s_{PPS} < s_{max}$$

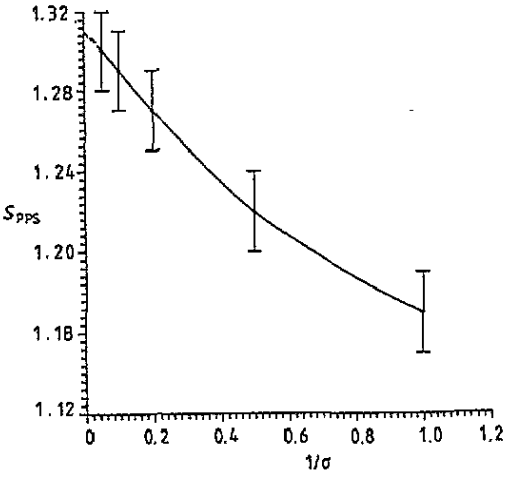

Figure 10. The dependence of s_{PPS}/ν on $1/\sigma_h$ (the inverse value of the host medium conductivity). The dashed line is an extrapolation to $\sigma_h = \infty$, which produces $s_{PPS}(0)/\nu = 1.31 \pm 0.02$.

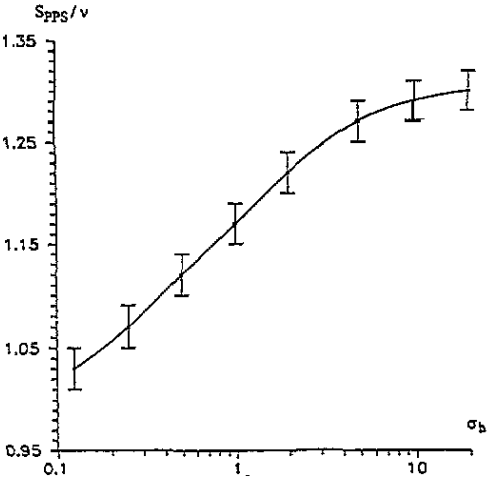

Figure 11. The dependence of s_{PPS}/ν on the conductivity σ_h of the host medium. The linear part of the plot agrees with the dependence that is obtained by means of RG consideration (see figure 14).

where $s_{max} = \nu(1.31 \pm 0.02) \simeq 1.41$ (figure 10).

Now we present some speculations in terms of RG evaluation to explain the result of the computer simulation. To take into account the additional configuration displayed in figure 7 we should introduce not only $g_h^{(n)}$ but also $\pi_h^{(n)}$ which is a conductance 'over a point' (see figure 12). In terms of the Bernasconi scheme the additional configurations appear only for $b > 2$. To simplify the consideration we confine ourselves to $b = 3$.

What values are $g_h^{(n)}$ and $\pi_h^{(n)}$ equal to? To answer this question we must consider the internal structure of a 'metallic' bond at the nth step of renormalization. It has a complicated structure. First, the superconducting cluster is not an intact spot but a complicated fractal,

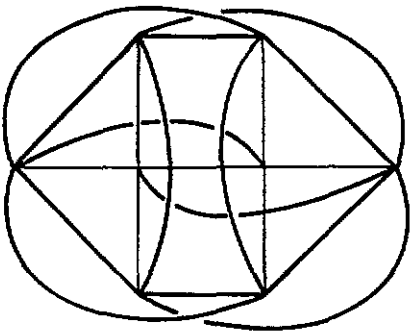

Figure 12. To take into account the additional configuration displayed in figure 7 we should introduce not only $g_h^{(n)}$ but also conductivity $\pi_h^{(n)}$ 'over the point'. In terms of the Bernasconi scheme the additional configuration appears only for $b > 2$. To simplify the consideration we confine ourselves to $b = 3$.

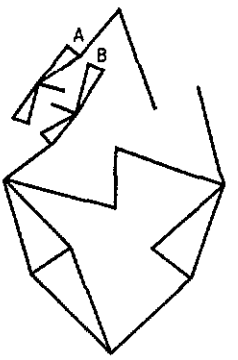

Figure 13. The scheme of the 'metallic' bond. Different parts of the bond are displayed with different resolution. It is seen that there are places separated by distance a_0.

almost each point of which may be a boundary point. So, $g_h^{(n)}$ is limited by $\sigma_h L^{d_f}$, where $d_f = 91/48 \simeq 1.895$ [5] is the fractal dimension of an infinite percolation cluster, and $L = b^n$. Second, the terminal of the dangling superconducting bond is not just a point but looks rather like a bunch. The distance between the ends of conducting bonds separated by a dielectric bond (points A and B in figure 13) is not equal to b^n but is about the infinitesimal gap a_0. The set of these gaps is responsible for the peculiarity of the dielectric constant at the percolation threshold. Thus both $g_h^{(n)}$ and $\sigma_{2D}^{(n)}$ are determined by the same topology: the tips of bunches separated by distance a_0 (see also [15]). Hence $g_h^{(n)}$ is at least proportional to $\sigma_{2D}^{(n)}$. It seems reasonable to expect that at $\sigma_h \ll \sigma_m$ the conductivity through the medium is determined by tips of bunches and $\pi_h^{(n)} \simeq \sigma_{PPS}^{(n)}\sigma_h/\sigma_m$. In the opposite limit $\sigma_h \gg \sigma_m$ not only the tips of the bunches but also all the parts of the effective bond contribute to $g_h^{(n)}$ and $\pi_h^{(n)}$, thus $\pi_h^{(n)} \simeq \sigma_h L^{d_f}$. Taking these cases as lower and upper estimations we use the scheme displayed in figure 8. Both cases yield $s_{PPS} > s_{2D}$ but with different dependence upon σ_h.

In the case $\sigma_h \ll \sigma_m$ it turns out that the properties of the PPS depend strongly upon the environment. The index s_{PPS} logarithmically depends upon the conductivity of the host medium (figure 14). Certainly $s_{PPS} = s_{2D}$ at $\sigma_h = 0$. The exponent s_{PPS} turns out to be not universal. In the case $\sigma_h \gg \sigma_m$ we obviously obtain $s_{PPS} = \nu d_f$. So the draft RG consideration produces the following bounds for s_{PPS}:

$$s_{2D} \leqslant s_{PPS} < \nu d_f.$$

The lower limit is achieved at $\sigma_h = 0$; the upper limit is achieved as $\sigma_h \to \infty$. This qualitatively agrees with the results of our simulation (figures 10 and 11).

Thus the PPS is a new non-universal system, the description of which demands that we take into account not only a layer with a percolation system but also the surrounding medium. In some sense we have non-local constitutive equations. The value and the scaling properties of σ_{PPS} depend on the nature of the medium at distances less than the correlation length $\xi_\perp$. This fact is very important for designing multilayer coatings with thin layers made of granular materials since we should use the constitutive parameters depending on the properties of neighbouring layers.

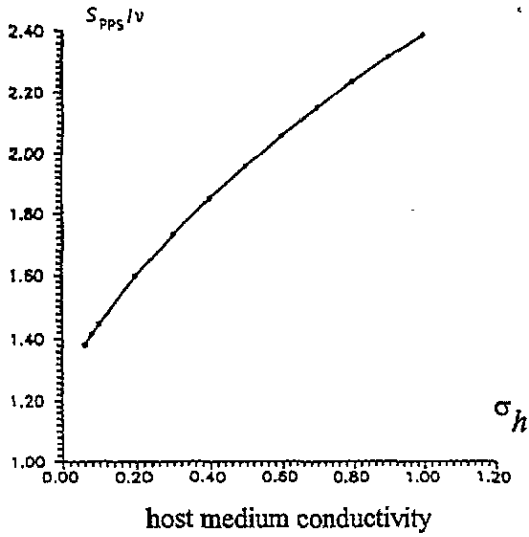

Figure 14. The dependence of the critical index s_{PPS}/ν on the conductivity of the host medium for $\pi_h^{(n)} \sim \sigma_{PPS}^{(n)}\sigma_h/\sigma_m$. The dependence is obtained in terms of RG considerations.

If we consider a thick sheet of granular material we have to take into account not only the cross-over from the 2D system to the 3D system [14] but also the effects described above. The impedance of the system would be determined by the value of ε_{PPS}.

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